EX NAVODAYAN FOUNDATION
(Registered Under Indian Trust Act 1882) Reg. No. : 2016, 43
B/36/43 46M Brahmanand Colony, Durgakund, Varanasi (UP) 221005
Ph. No. : 7607007013, 7607005199 Email Id : exnavodayanfoundation@gmail.com

1st Online Practice Test XII-IIT Solution & Answer - Key

Physics

1. (a) Charge on the inner surface is –q consider charge a on outer surface of B. Then i Potential at outer surface of B is zero (as earthed)

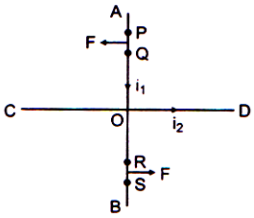
$$\frac{-q}{4\pi\in_0 C}+\frac{q'}{4\pi\in_0 b}=0$$

$$a'=\left(\frac{b}{c}\right)a$$

2. (a)
Applying Ampere's law to the rectangle shown in figure :

$(2Bl)=\mu_0(\lambda l)$

$\therefore \quad B=\frac{\mu_0\lambda}{2}$



3. (c) Given $V(x_1y_1z_1)=A\times y-Bx^2+cy.$

$$E_z=\frac{OV}{O\notin}$$

$$= -\frac{0(A\times y-Bx^2+cy)}{0\notin}$$

$= 0$

4. (b) Given electric field $\vec{E}=(5\hat{i}+3\hat{j})kv/m$

and $r_A=(o\hat{i}+o\hat{j}+o\hat{k})$

$r_B=(4\hat{i}+o\hat{j}+3\hat{k})\Rightarrow r_{BA}=(4\hat{i}+3\hat{k})$

Now

$V_B-V_A=-\vec{E}.\vec{r}_{BA}$ [for constant $\vec{E}$]

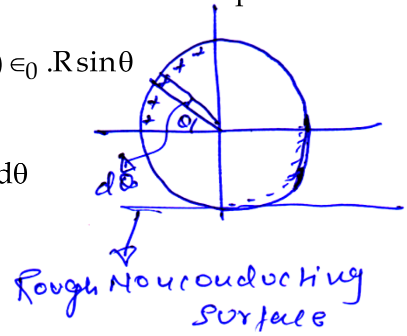
$V_B-V_A=-(5\hat{i}-3\hat{j}).(4\hat{i}+3\hat{k})$

$\boxed{V_B-V_A=-20kv}$

5. (d)
Here, torque produced by positive charge and negative charge distribution are equal.

$$\int_0^2 dz=\int_0^{\pi/2}(\lambda Rd\theta)\in_0 .R\sin\theta$$

$$=\lambda E_0R_0^2\int^{\pi/2}\sin\theta d\theta$$



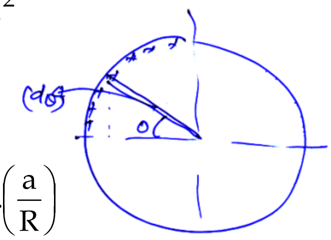
$=\lambda E_0R^2$ and $=\lambda E_0R^2$

Torque = $2\lambda E_0R^2$

Now , $f = ma$

and

$$2\lambda E_0R^2-fR=mR^2.\left(\frac{a}{R}\right)$$



Solving. f = λE_0R along positive x-axis.

6. (d) A = (2, 2) and B = (4, 1)

Now $W_{A\to B}=1(V_B-V_A)$ ……..(1)

$$\int^B dV=-\int_A^B \vec{E}.\vec{dr}$$

or $V_B-V_A=-\int_{(2,2)}^{(4,1)}(y\hat{i}+x\hat{j}).(dx\hat{i}+dy\hat{j}+dz\hat{k})$

or $V_B-V_A=-\int_{(2,2)}^{(4,1)}(ydx+xdy)$

$$=-\int_{(2,2)}^{(4,1)}d(xy)=[-xy]_{2,2}^{4,1}=0$$

$\therefore \ W_{A\to B}=0$

7. (d)
When either A or C is earthed (but not both together), a parallel plate capacitor is forced with B, with +Q charged on the inner surfaces. [The other plate, which is not earthed, plays no role.] Hence, charge of amount +Q flows to earth.
When both are earthed together, A and C effectively become connected. The plates now form two capacitors in parallel, with capacitances in the ratio 1 : 2 and hence, share charge Q in the same ratio.

8. (d)
Since, the capacitor plates are directly connected to the battery, it will take no time in charging.

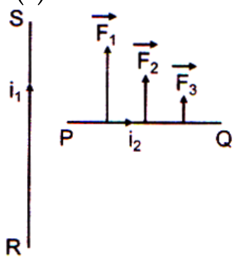
9. (c)
In case of a capacitor
q = CV

$\therefore\ i = \left(\frac{dq}{dt}\right) = C\left(\frac{dV}{dt}\right)$

$\frac{dV}{dt} = \frac{4.0}{4.0} V/s = 1.0\ V/s$

Therefore, it C = 1 F
then i = 1 × 1 = 1 A (constant)

10. (c)



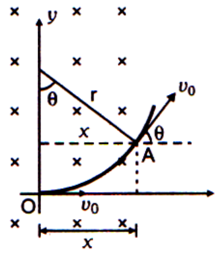
Every current carrying loop is a magnetic dipole. If it lies in the plane of paper and current in it is in clockwise direction magnetic field at all points lying within the loop is perpendicular to paper inwards and at points outside the loop magnetic field is perpendicular to paper in outward direction. For $\theta < 180^o$, the centre O lies outside the loop and current is clockwise. Therefore magnetic field is perpendicular to paper in outward direction.

11. (a)

12. (d) Forces acting on the loop due to i_1 act in the plane of the loop giving zero torque.

13. (b)
$\vec{F}_{AOB} = \vec{F}_{AB} = i(\vec{I} \times \vec{B})$
Here, AB = $2\sqrt{2 \times 2} = 4m$
$\therefore\ \vec{F}_{AB} = 2[(-4\hat{j}) \times (-4\hat{j})] = 32\hat{i}$

14. (c) $r = \frac{mv_0}{B_0 q} = \frac{v_0}{B_0 \alpha}$



$\frac{x}{r} = \frac{\sqrt{3}}{2} = \sin\theta$

$\therefore\ \theta = 60^o$

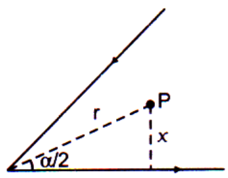
$t_{OA} = \frac{T}{6} = \frac{\pi}{3B_0\alpha}$

Therefore,
x-co-ordinate of particle at any time
$t > \frac{\pi}{3B_0\alpha}$ will be

$x = \frac{\sqrt{3}}{2}\frac{v_0}{B_0\alpha} v_0\left(t - \frac{\pi}{3B_0\alpha}\right)\cos 60^o$

$= \frac{\sqrt{3}}{2}\frac{v_0}{B_0\alpha} + \frac{v_0}{2}\left(t - \frac{\pi}{3B_0\alpha}\right)$

15. (c)
$x = r\sin\frac{\alpha}{2}$



$\therefore\ B_P = 2\left(\frac{\mu_0}{4\pi}\right)\left(\frac{i}{x}\right)\left[\sin\left(90^\circ - \frac{\alpha}{2}\right) + \sin 90^\circ\right]$

$= \frac{\mu_0}{2\pi}\frac{i}{r}\frac{\left(1 + \cos\frac{\alpha}{2}\right)}{\sin\frac{\alpha}{2}}$

16. (c)
$U = \frac{1}{2}CV^2 = 0.15 mJ$
This energy will be dissipated in the resistors in the ratio of their resistances.
$\therefore\ H_{4\Omega} = \left(\frac{4}{6}\right)(0.15) = 0.1 mJ$

17. (a)
Under steady state condition, power developed = power lost
or = $i^2R = \lambda(\theta - \theta_0)$
or $(\theta - \theta_0) = \frac{i^2R}{\lambda}$

This is also the increase in temperature of the resistance.

$\therefore\ \Delta l = l\alpha\Delta\theta$

or strain $= \dfrac{\Delta l}{l} = \alpha\Delta\theta$

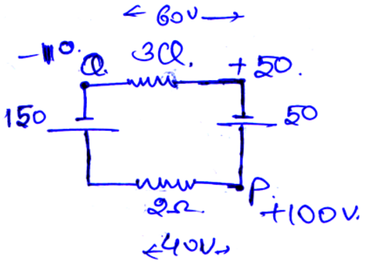
$= \alpha(\theta-\theta_0) = \dfrac{i^2R\alpha}{\lambda}$

Note : Initially the temperature of resistance will be equal to the temperature of the atmosphere.

18. (b)

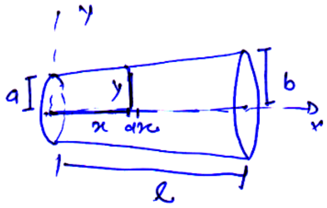
$V_{net} = 100V,$

$R_{net} = 5, I = 20A$



19. (a) We take a differential element (dx) at x. as shown in figure.

Then



$y = \left(\dfrac{b-a}{e}\right)x + a$

Now;

$$\int_0^R dR = \int_0^\ell \dfrac{\int dx}{x\left[\dfrac{(b-a)}{e}x+a\right]^2}$$

We get ;

$$\boxed{R = \dfrac{\rho\ell}{\pi ab}}$$

20. (d)

From conservation of mechanical energy

$$\frac{1}{2}kr^2 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q^2}{r} - \frac{q^2}{r+r}\right]$$

or $\dfrac{1}{2}kr^2 = \dfrac{q^2}{8\pi\varepsilon_0 r}$

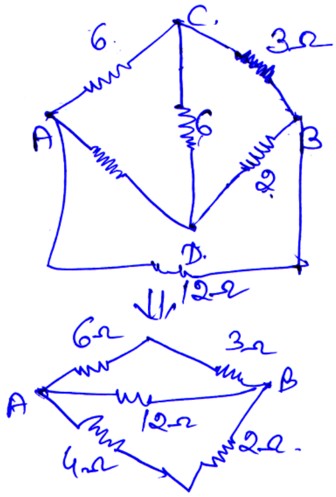
$\therefore\ k = \dfrac{q^2}{4\pi\varepsilon_0 r^3}$

21. (c)

$$\frac{1}{Req} = \frac{1}{9} + \frac{1}{6} + \frac{1}{12} = \frac{4+6+3}{36} = \frac{13}{36}$$

$$\frac{1}{Req} = \frac{13}{36} \Rightarrow Req = \frac{36}{13}$$

$$I = \frac{V}{Req} = \frac{12}{36} = \frac{13}{3}A$$



22. (c) $R_A = 2\Omega, R_V = 200\Omega$

$R = 50\Omega, r = 1\Omega$

$\varepsilon = 4.3V$

$$R_{net} = r + \frac{(R+R_A)R_V}{R+R_A+R_V}$$

$$= 1 + \frac{52\times200}{252} = \frac{252+10400}{252}$$

$$R_{net} = \frac{10652}{252}$$

$$I = \frac{\varepsilon}{R_{net}} = \frac{4.3\times252}{10652}A$$

$$I_1 = \left[\frac{R_V}{(R+R_A)+R_V}\right]\times I$$

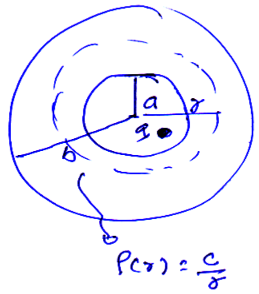
$$= \frac{200}{252}\times\frac{4.3\times252}{10652}$$

$$= \frac{860}{10652} = 0.08A$$

23. (b)

We take a gassian surface at (r)

Charge at (r)



Charge enclosed by the surface

$$= q_0 + \int_a^r 4\pi r^2 dr.\frac{c}{r}$$

$= q_0 + \frac{4\pi cr^2}{2}$

$= q_0 + \frac{4\pi c}{2}(r^2 - a^2)$

Now,

$\phi = \frac{qin}{\varepsilon_0}$

$E.4\pi r^2 = \frac{q_0}{\varepsilon_0} + \frac{4\pi C}{2\varepsilon_0}\left(r^2 - a^2\right)$

$E = \left(\frac{r^2 - a^2}{2E_0 r^2}\right)c + \frac{q_0}{4\pi E_0 r^2}$

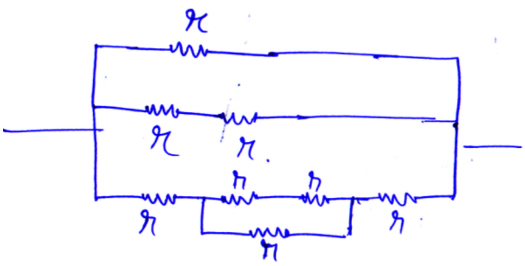
$= \frac{C}{2E_0} - \frac{a^2 C}{2E_0 r^2} + \frac{q_0}{4\pi E_0 r^2}$

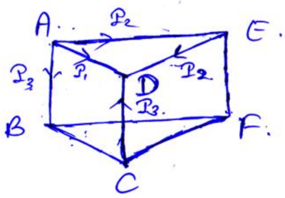
$= \frac{C}{2E_0} + \frac{I}{r^2}\left(\frac{q_0}{4\pi E_0} - \frac{a^2 6}{2E_0}\right)$

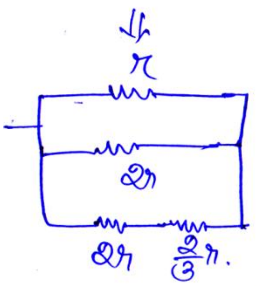
Making coefficient $\frac{1}{r^2} \to$ zero

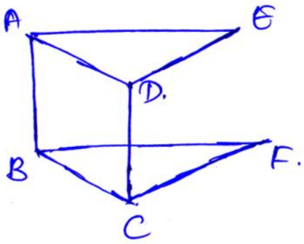
$\boxed{C = \frac{q_0}{2\pi a^2}} \Rightarrow \boxed{n = 2}$

24. (a)







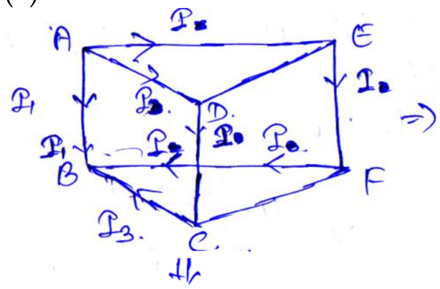


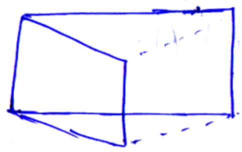
$\Rightarrow \frac{1}{Req} = \frac{1}{r} + \frac{1}{2r} + \frac{3}{8r}$

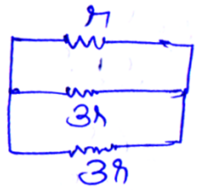
$\frac{1}{Req} = \frac{8+4+3}{8r}$

$\Rightarrow Req = \frac{8r}{15}$

(b)





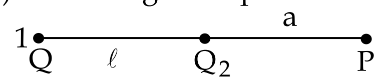


$\frac{1}{Req} = \frac{1}{r} + \frac{1}{3r} + \frac{1}{3r}$

$= \frac{3+1+1}{3r} = \frac{5}{3r}$ Req $= \frac{3}{5}r$

25. (b)

26. (a) Two charges are placed as shown in figure



from graph electric field at point P is zero.

$\frac{Q_1}{4\pi E_0(\ell + a)^2} + \frac{Q_2}{4\pi E_0 a^2} = 0 \Rightarrow \left|\frac{Q_1}{Q_2}\right| = \left(\frac{\ell + a}{a}\right)^2$

27. (c)

Let C be the capacity without slab. Then after removing the slab, net capacity is $\frac{C}{2}$.

Before the slab is removed

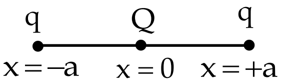
$C_1 = C$ and $C_2 = KC$

$\therefore C_{net} = \left(\frac{K}{K+1}\right)C$

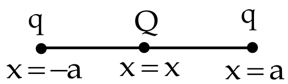
$\therefore Q_1 = Q_2 = \left(\frac{KCE}{K+1}\right)$

$\therefore \frac{Q'_2}{Q_2} = \frac{K+1}{2K}$

28. (b)



Initial position



Final position

$$U_i = \frac{2KQq}{d}$$

and $U_f = KQq\left[\frac{1}{a+x} + \frac{1}{a-x}\right]$

Here, $K = \frac{1}{4\pi\varepsilon_0}$

$\Delta U = U_f - U_i$

or $|\Delta U| = \frac{2KQqx^2}{a^3}$ for $x << a$

$\therefore\ \Delta U \propto x^2$

29. (a)

$$\frac{\text{Magnetic moment}}{\text{Angular momentum}} = \frac{q}{2m}$$

$\therefore$ Magnetic moment = (angular momentum) $\frac{q}{2m}$

$= (I\omega)\frac{q}{2m}$

$= \left(\frac{1}{2}mR^2\right)(\omega)\left(\frac{q}{2m}\right)$

$= \frac{1}{4}q\omega R^2$

30. (c)
Since, the magnetic field due to wire RS is non uniform. Neither the force nor the torque on wire PQ is zero. Therefore, it will have both translational as well as rotational motion.

Chemistry

1. (b) $P_A = X_A P_A^o = Y_A P_{Total}$

$P_{Total} = X_A P_A^o + (1 - X_A)P_B^o$

So, $X_A P_A^o = Y_A\left[P_B^o + X_A(P_A^o - P_B^o)\right]$

$$\frac{1}{Y_A}\left(\frac{P_A^o}{P_B^o}\right) = \frac{1}{X_A}\left[1 + X_A\frac{(P_A^o - P_B^o)}{P_B^o}\right]$$

$$\frac{1}{X_A} = \frac{1}{Y_A}\left(\frac{P_A^o}{P_B^o}\right) + \frac{(P_B^o - P_A^o)}{P_B^o}$$

Therefore,

slope = $\frac{P_A^o}{P_B^o}$; Intercept = $\frac{P_B^o - P_A^o}{P_B^o}$

2. (c) Instantaneous rate of reaction, R

$$R = -\frac{1}{2}\frac{d[HI]}{dt} = \frac{d[H_2]}{dt} = \frac{d[I_2]}{dt}$$

3. (d) $Fe^{+3} + e \longrightarrow Fe^{+2}$ will be obtained from (2) – (1)

So, $\Delta G = \Delta G_2 - \Delta G_1$

$-1FE = -3F(-0.036V) + 2F(-0.440)$

Solving, $E = +0.772V$

4. (c) If X % occupied density is – 0.96g/cc
For, 100% occupied (No empty space) d = $\frac{0.96}{X}100 = \frac{96}{x}$

% Empty space = $\frac{\frac{96}{x} - 0.99}{\frac{96}{x}}100 =$

$\left[100 - \frac{33}{32}\times\right]$

5. (c) $S = k_H P$

$\frac{m}{1} = k_H(1)$ .... (1)

$\frac{m'}{2} = k_H(5)$ .... (2)

From (1) and (2)
$m' = 10m$

6. (b) $k = \frac{10^{-2}\,lit}{mol\ \ sec}$

$$\frac{10^{-2}\times 10^3 ml \times 60}{6.023\times 10^{23}\ molecules \times 1 min} \approx 10^{-21}$$

7. (b) Physical adsorption is an exothermic process. So it decreases with increase in temperature
8. (a) No change in density.
9. (d) Let x moles of A is in vapour state.
So, In vapour state moles are –
A = $10 - x$
B = x
If P is required pressure. Then using Rault's law and Dalton's law –

$P_A = \frac{x}{10}P = \frac{10-x}{10}200$

$P_B = \frac{10-x}{10}P = \frac{x}{10}100$

$x = \frac{10P}{P+100} = \frac{2000}{P+200}$

$\Rightarrow P = 141.42$ torr.

10. (c) Rate = $K[A]^o = K = \text{constant}$.

11. (c) Catalyst do not alter state functions.

12. (d)

13. (d) Non-volatile solute decreases surface covered by solvent and hence decreases vapourisation.

14. (d) At Anode,

$Ag(s) \longrightarrow Ag^+_{(aq)} + e$

At cathode,

$Ag^+ + e \longrightarrow Ag(s)$

0.10

$E_{cell} = E^o_{cell} - \frac{0.0591}{1}\log\left(\frac{x}{0.10}\right)$

0.413 V = O – 0.0591 $[\log x - \log 10^{-1}]$

$K_{SP} = x^2$ $\qquad x = 10^{-8}$

15. (b) Protective power $\propto \frac{1}{\text{Gold number}}$

16. (b) The common ion is preferential absorb. So in $Fe(OH)_3$ and $FeCl_3$, common ion is Fe^{3+} ions.

17. (d) For NaCl, Z = 1

$d = \frac{Z \times \text{Atomic mass}}{N_A \times a^3} =$

$\frac{4 \times 58.5}{6.022 \times 10^{23} \times 4.7 \times 10^{-23}}$ = 0.12 ml

18. (b) π = i C S T

$= 2 \times \frac{w\,ST}{v.m} = 2 \times \frac{2.5 \times 1000 \times 0.0821 \times 300}{100 \times 58.5} =$

21 atm

19. (b) $3A + 2B \longrightarrow \text{Product}$

rate = $k[A][B]^2$

If A is large excess then (A) not considered so rate = $k[B]^2 \longrightarrow$ 2nd order kinetics

20. (c) In ccp (i) C occupied corners and face centres

(ii) Tetrahedral voids present at four diagonally position in shaded plane.

(iii) Octahedral voids is present at body centre and edge centres.

21. (c) $\Delta T_f = i.\,k_f.m \Rightarrow 3.72 = i \times 1.86 \times 1$

$[Pt(H_2O)_3Cl_3]Cl \longrightarrow [Pt(H_2O)_3Cl_3]^+ + Cl^-$

22. (a) $E_{cell} = E^o_{cell} - \frac{0.0591}{n}\log\frac{(Ni^{++})}{(Cu^{++})}$

$[Ni^{++}]$ ↓es then E_{cell} ↑es.

23. (d) $Fe(OH)_3$ is positive sol so $-$ive ions is more effective for coagulation. Greater the magnitude of $-$ive charge, greater will be coagulation.

$K_3(Fe(CH)_6) \longrightarrow 3K^+ + [Fe(CN)_3]^{3-}$

24. (d) $Cl_2 \underset{}{\overset{K_1}{\rightleftharpoons}} 2Cl^\bullet$ (fast)

$K_{eq} = \frac{K_1}{K_2} = \frac{K_2[Cl^\bullet]^2}{[Cl_2]} \Rightarrow [Cl^\bullet] = K_{eq}^{\frac{1}{2}}(Cl_2)^{\frac{1}{2}}$

$CHCl_3 + Cl^\bullet \xrightarrow{K_3} HCl + \dot{C}Cl_3$ (slow) (rate determining step)

rate = $K_3[Cl^\bullet][CHCl_3] = K_3K_{eq}^{\frac{1}{2}}(Cl_2)^{\frac{1}{2}}(CHCl_3)$

25. (b) $N = N_o e^{-\frac{\Delta H}{RT}}$

$1 = 10^{10} e^{-\frac{\Delta H}{R(1100)}}$ ...... (1)

$1 = 2 \times 10^9 e^{-\frac{\Delta H}{R(1200)}}$ .... (2)

1 divided (2)

$1 = 5e^{\frac{\Delta H}{R}\left(\frac{1}{1200} - \frac{1}{1100}\right)}$

$\frac{1}{5} = e^{\frac{\Delta H}{R}\left(\frac{-100}{1100 \times 1200}\right)}$

$-2.303\log 5 = \frac{\Delta H}{8.314}\left(-\frac{100}{1100 \times 1200}\right)$

$\Delta H = \frac{2.303 \times 0.699 \times 8.314 \times 1100 \times 1200}{100} \times 10^{-3}\,kJ/mole$

= 176.8 kJ/mole

26. (a) $X + Y = 0.1$

	x mole KCl	2y mole $BaCl_2$
	↓	↓
So	$i = 2$	$i = 3$

For observed range

$y = 0.1 - x$	$x = 0.1 - y$
$3y = 3(0.1 - x)$	$2x = 0.2 - 0.2y$
$= 0.3 - 3x$	if $y = 0$
if $x = 0$	then 2x = 0.2

then $3y = 0.3$

$\Delta T_f = 1.86 \times .3$ $\Delta T_f = 1.86 \times 0.2$

$= 0.558^oC$ $= 0.372^oC$

So observed range is $.372^oC$ to 0.558^oC

27. (c) Least CMC value $\propto$ long chain length.

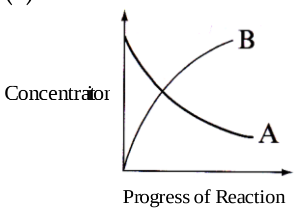
So ans in $C_{11}H_{23}NH_4Cl$

28. (b) Milliequivalent of A = Melliequivalent of B

$$\frac{\frac{5.6}{112}}{x} \times 1000 = \frac{\frac{0.9}{27}}{3} \times 1000$$

x = 2

29. (c)



$A \longrightarrow nB$

$A_o(1-\alpha)$ $A_o n\alpha$

At the point of intersection

$[A] = [B]$

$[1-\alpha] = A_o n\alpha$

$1 = \alpha(n+1)$

$\alpha = \frac{1}{n+1}$

So $[B] = A_o\ n\alpha = \frac{A_o n}{n+1}$

30. (a) XY_2O_4

Since number of O^{2-} = 4

Number of octahedral void = 4

One octahedral void occupied by x

One octahedral void occupied by y

So, Fraction of octahedral void

occupied = $\frac{2}{4} = \frac{1}{2}$

Mathematics

1. Sol.(b)

We have $f(x) = \frac{\sqrt{3-x}}{(x-1)\,(x-2)(x-3)} + \sin^{-1}\left[\frac{3x-2}{2}\right]$ $f(x)$ is defined if

(i) $-1 \le \left[\frac{3x-2}{2}\right] \le 1$

$\Rightarrow -1 \le \frac{3x-2}{2} < 2 \Rightarrow -2 \le 3x-2 < 4 \Rightarrow 0 \le 3x < 6 \Rightarrow 0 \le x < 2$ (i)

(ii) $3 - x \ge 0$ and $x \ne 1, 2, 3 \Rightarrow x < 3$ and $x \ne 1,2$ (ii)

From (i) and (ii),

$x \in [0,2) - \{1\}$

2. Sol.(a)

$$\Delta = \begin{vmatrix} a_1 + b_1 w & a_1 w^2 + b_1 & c_1 + b_1 \overline{w} \\ a_2 + b_2 w & a_2 w^2 + b_2 & c_2 + b_2 \overline{w} \\ a_3 + b_3 w & a_3 w^2 + b_3 & c_3 + b_3 \overline{w} \end{vmatrix}$$

Using $C_2 \to wC_2$, we get

$$\Delta = \frac{1}{w}\begin{vmatrix} a_1 + b_1 w & a_1 w^3 + b_1 w & c_1 + b_1 \overline{w} \\ a_2 + b_2 w & a_2 w^3 + b_2 w & c_2 + b_2 \overline{w} \\ a_3 + b_3 w & a_3 w^3 + b_3 w & c_3 + b_3 \overline{w} \end{vmatrix} = \frac{1}{w}\begin{vmatrix} a_1 + b_1 w & a_1 + b_1 w & c_1 + b_1 \overline{w} \\ a_2 + b_2 w & a_2 + b_1 w & c_2 + b_2 \overline{w} \\ a_3 + b_3 w & a_3 + b_3 w & c_3 + b_3 \overline{w} \end{vmatrix} = 0$$

3. Sol.(c)

Given , $y = x^{\log_x \pi} = \pi$

Domain $= x \in (0,1) \cup (1, \infty)$

Range = $\{\pi\}$

4. Sol.(d)

$B = A^{2^n} = A^{2 \cdot 2^{n-1}} = (A^2)^{2^{n-1}} = (A^{-1})^{2^{n-1}} = (A^{2^{n-1}})^{-1} = (A^{2.2^{n-2}})^{-1} = ((A^2)^{2^{n-2}})^{-1} = ((A^{-1})^{-1})^{2^{n-2}}$

$= A^{2^{(n-2)}}$

So, $B = C \Rightarrow (B - C) = O \Rightarrow \det.(B - C) = 0$

5. Sol.(b)

Total number of third order determinants is 9 ! .

Since number of determinants is even and in these there are $\frac{9!}{2}$ pairs of determinants which are obtained by changing two consecutive rows.

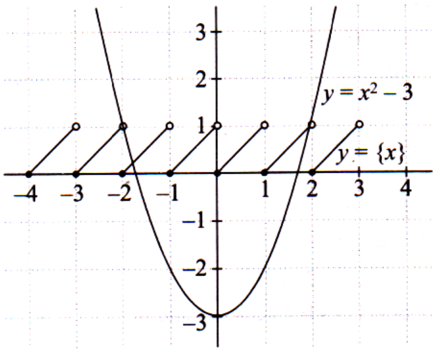
So, $\sum_{i=1}^{n} D_i = 0$

6. Sol.(d)

Let $f(x) = x^2 - 3$ and $g(x) = \{x\}$.

Clearly , $f(x) = g(x)$ only if $-2 < x < -1$

$\therefore \ [x] = -2$



Given equation becomes

$x^2 - 3 - (x - [x]) = 0$

$x^2 - 3 - x + (-2) = 0$

$x^2 - x - 5 = 0$

$x = \frac{1 - \sqrt{21}}{2}$

7. Sol.(a)

Given $AB + A + B = 0$

$\Rightarrow AB + A + B + I = I \Rightarrow A(B + I) + (B + I) = I \Rightarrow (A + I)(B + I) = I$

$\Rightarrow (A + I)$ and $(B + I)$ are inverse of each other

$\Rightarrow (A + I)(B + I) = (B + I)(A + I) \Rightarrow AB = BA$

Thus , A and B are commutative

$\Rightarrow (A + B)^2 = A^2 + 2AB + B^2$

8. Sol.(c)

$f(x) = f(-x)$ where $f(x) = x|x| + \sin|x| + xe^x$,

$\therefore F(x) = x^2 - \sin x - xe^{-x}$

Also , $g(x) = -g(-x)$, where $g(x) = \cos x + x^2 - x$

$\therefore \ G(x) = -(\cos x + x^2 + x) = -\cos x - x^2 - x$

$\therefore \ F(x) + G(x) = = -\sin x - xe^{-x} - \cos x - x = -(\sin x + \cos x + x + xe^{-x})$

9. Sol.(b)

$$S_n=\sum_{r=1}^{n}\tan^{-1}\left(\frac{2(2r-1)}{4+r^2(r^2-2r+1)}\right)=\sum_{r=1}^{n}\tan^{-1}\left(\frac{2(2r-1)}{4+r^2(r-1)^2}\right)=\sum_{r=1}^{n}\tan^{-1}\left(\frac{\frac{2r-1}{2}}{1+\frac{r^2}{2}.\frac{(r-1)^2}{2}}\right)$$

$$=\sum_{r=1}^{n}\tan^{-1}\left(\frac{\frac{r^2}{2}-\frac{(r-1)^2}{2}}{1+\frac{r^2}{2}.\frac{(r-1)^2}{2}}\right)=\sum_{r=1}^{n}\left(\tan^{-1}\frac{r^2}{2}-\tan^{-1}\frac{(r-1)^2}{2}\right)=\tan^{-1}\frac{n^2}{2}$$

$\therefore\ S_\infty=\pi/2$

10. Sol.(a)

If $f(x)$ is surjective then range of $f(x)$ must be $[1,\infty)$.

$\therefore$ Range of $\sqrt{3x^2-4x+k+1}+10\in[10,\infty)$

$\Rightarrow$ Range of $3x^2-4x+k+1\in[0,\infty)\Rightarrow D=0\Rightarrow 16-12(k+1)=0\Rightarrow 4-3k-3=0\Rightarrow k=\frac{1}{3}$

11. Sol.(b) Let $t=\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

$\Rightarrow\frac{-\pi}{3}<t-\frac{\pi}{6}<\frac{\pi}{3}\Rightarrow\frac{-\pi}{6}<t<\frac{\pi}{2}\Rightarrow 0\le t<\frac{\pi}{2}\Rightarrow 0<\frac{1-x^2}{1+x^2}\le 1$

$\Rightarrow\ 0<1-x^2\le 1+x^2\Rightarrow 0\le x^2<1\Rightarrow x\in(-1,1)$

12. Sol.(c)

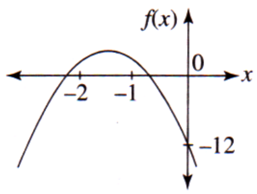
$g(x)\,ke[-2,-1)\ \forall\, x\in R$

$f(x)=-3x^2-kx-12$

$\Rightarrow f(0)=-12$

$\because\ f(g(x))>0\ \forall\ x\in R$

$\therefore\ f(-2)>0$ and $f(-1)\ge 0$



13. Sol.(a) $y=(\sin^{-1}(\sin x))^2-\sin^{-1}(\sin x)=\left(\sin^{-1}(\sin x)-\frac{1}{2}\right)^2-\frac{1}{4}$

For maximum value of y, $\sin^{-1}(\sin x)=-\frac{\pi}{2}\Rightarrow y=\left(\frac{\pi}{2}+\frac{1}{2}\right)^2-\frac{1}{4}=\frac{\pi}{4}(\pi+2)$

14. Sol.(a) $L=\lim\limits_{x\to-7}\frac{[x]^2+15[x]+56}{\sin(x+7)\sin(x+8)}=\lim\limits_{x\to-7}\frac{([x]+7)([x]+8)}{\sin(x+7)\sin(x+8)}$

Now, $\lim\limits_{x\to-7^+}\frac{([x]+7)([x]+8)}{\sin(x+7)\sin(x+8)}=\frac{(exact\ 0).(1)}{(\to 0)\sin(-1)}=0$

$\lim\limits_{x\to-7}\frac{([x]+7)([x]+8)}{\sin(x+7)\sin(x+8)}=\frac{(-1).(exact\ 0)}{(\to 0)\sin(-1)}=6$

$\therefore\ L=0$

15. Sol.(c) $f(x)=\sqrt{\frac{\pi}{2}-\tan^{-1}\sqrt{-x^2+5x-6}}$

For domain of function , $\frac{\pi}{2}-\tan^{-1}\sqrt{-x^2+5x-6}\ge 0$

$\Rightarrow\ \tan^{-1}\sqrt{-x^2+5x-6}<\frac{\pi}{2}\Rightarrow -x^2+5x-6\ge 0\Rightarrow x^2-5x+6\le 0\Rightarrow x\in[2,3]$

Therefore, integral values of x are { 2, 3}

16. Sol.(c) We must have $-1 \le \frac{4\tan^{-1}(2\pi x)}{\pi} \le 1$

$\Rightarrow -\frac{\pi}{4} \le \tan^{-1} 2\pi x \le \frac{\pi}{4} \Rightarrow -1 \le 2\pi x \le 1 \Rightarrow x \in \left[-\frac{1}{2\pi}, \frac{1}{2\pi}\right]$

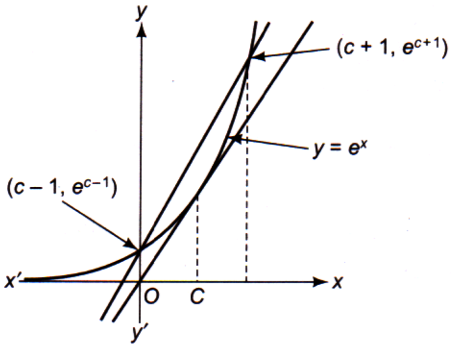
17. Sol(a) Slope of the tangent to $y = e^x$ at (c, e^c) is given by $m_1 = \left(\frac{dy}{dx}\right)_{(c,e^c)} = e^c$

Also , slope of the line joining the points $(c-1, e^{c-1})$ and $(c+1, e^{c+1})$ is

$$m_2 = \frac{e^{c+1} - e^{c-1}}{(c+1)-(c-1)} = \frac{e^{c+1} - e^{c-1}}{2} = e^c\left(\frac{e - e^{-1}}{2}\right)$$

We observe $m_2 > m_1$

Thus, tangent to the curve $y = e^x$ at (c, e^c) will intersect the given line to the left of the line $x = c$ as shown in the figure.



18. Sol.(b) $\lim_{x\to\infty} (e^x + \pi^x)^{1/x} = \pi \lim_{x\to\infty}\left(\left(\frac{e}{\pi}\right)^x + 1\right)^{1/x} = \pi$

Now, $\{\pi\} = \pi - 3$

$\therefore \lim_{x\to\infty}\left\{(e^x + \pi^x)^{\frac{1}{x}}\right\} = \pi - 3$

19. Sol.(c) $f(x) = x - 1, 1 \le x \le 2$

$g(x) = x - 1 = b\sin\frac{\pi}{2}x, 1 \le x \le 2$

$f(1) = 0;\ f(2) = 1 \Rightarrow$ Rolle's theorem is not applicable to 'f but LMVT is applicable to f

($\because$ $x-1$ is continuous and differentiable in [1, 2] and (1,2) respectively)

Now $g(1) = b;\ g(2) = 1$ and

Function $x-1$, $\sin\frac{\pi}{2}x$ are both continuous in [1,2] and (1,2)

$\therefore$ For Rolle's theorem to be applicable to g, we must have b = 1

20. Sol.(c)

Let $L = \lim_{x\to 0}\frac{e^{x^2} - e^x + x}{1-\cos 2x} = \lim_{x\to 0}\frac{e^{x^2} - 1 + x - e^x + 1}{2\frac{\sin^2 x}{x^2}.x^2} = \frac{1}{2}\left[\lim_{x\to 0}\frac{e^{x^2}-1}{x^2} + \lim_{x\to 0}\frac{x - e^x + 1}{x^2}\right]$

$= \frac{1}{2}\left[1 - \lim_{x\to 0}\frac{e^x - 1}{2x}\right]$ (using L' Hospital Rule) $\quad = \frac{1}{2}\left[1 - \frac{1}{2}\right] = \frac{1}{4}$

21. Sol.(d)

$y=3x+3$ is normal to the curve at $x=0$

Slope of normal is 3.

$\therefore$ Slope of tangent at $x=0$ is $f'(0)=-\frac{1}{3}$

Now $\lim_{x\to 0}\frac{x^2}{\{f(x^2)-5f(4x^2)+4f(7x^2)\}}\left(\frac{0}{0} \text{ from}\right)$

$=\lim_{x\to 0}\frac{2x}{2xf'(x^2)-40xf'(4x^2)+56xf'(7x^2)}=\lim_{x\to 0}\frac{1}{f'(0)-20f'(0)+28f'(0)}=\frac{1}{9f'(0)}=\frac{1}{9\left(-\frac{1}{3}\right)}=-\frac{1}{3}$

22. Sol.(b)

$\lim_{x\to 0}\frac{x^3}{\sqrt{a+x}(bx-\sin x)}=1\Rightarrow \lim_{x\to 0}\frac{x^3}{\sqrt{a}.(bx-\sin x)}=1\Rightarrow \lim_{h\to 0}\frac{3x^2}{\sqrt{a}(b-\cos x)}=1$

$\therefore\ b=1$

$\therefore\ 1=\lim_{x\to 0}\frac{3x^2}{\sqrt{a}(1-\cos x)}=\lim_{x\to 0}\frac{3x^2}{\sqrt{a}\,2\sin^2\frac{x}{2}}=\frac{6}{\sqrt{a}}$

$\therefore\ a=36$

Thus $a+b=37$

23. Sol.(a)

In the nbd of $x=7\pi/6$, we have $f(x)=|\sin x+\cos x|=-\sin x-\cos x$

$\Rightarrow\ f'(x)=-\cos x+\sin x\Rightarrow f'(7\pi/6)=-\cos(7\pi/6)+\sin(7\pi/6)=\frac{\sqrt{3}-1}{2}$

24. Sol.(b)

As $f(x)$ is continuous for all $x\in R$.

Thus, $\lim_{x\to\sqrt{3}} f(x)=f(\sqrt{3})$

Where $f(x)=\frac{x^2-2x+2\sqrt{3}-3}{\sqrt{3}-x}, x\neq\sqrt{3}$

Now $\lim_{x\to\sqrt{3}} f(x)=\lim_{x\to\sqrt{3}}\frac{x^2-2x+2\sqrt{3}-3}{\sqrt{3}-x}=\lim_{x\to\sqrt{3}}\frac{(2-\sqrt{3}-x)(\sqrt{3}-x)}{(\sqrt{3}-x)}=2(1-\sqrt{3})$

$\Rightarrow\ f(\sqrt{3})=2(1-\sqrt{3})$

25. Sol.(d)

$\sin(x+y)=\log_e(x+y)$

$\cos(x+y)[1+y']=\frac{1}{x+y}(1+y')\Rightarrow y'\left(\cos(x+y)-\frac{1}{x+y}\right)=\frac{1}{x+y}-\cos(x+y)\Rightarrow\frac{dy}{dx}=-1$

26. Sol.(c)

(i) $f(x)=\begin{cases}1, & x\le 0\\ -1, & x>0\end{cases}$

(ii) $f(x)=\begin{cases}1, & x<0\\ -1, & x\ge 0\end{cases}$

(iii) $f(x)=\begin{cases}-1, & x\le 0\\ 1, & x>0\end{cases}$

(iv) $f(x)=\begin{cases}-1, & x<0\\ 1, & x\ge 0\end{cases}$

(v) $f(x)=\begin{cases}1, & x>0\\ 1, & x<0\\ -1, & x=0\end{cases}$

(vi) $f(x)=\begin{cases}-1, & x>0\\ -1, & x<0\\ 1, & x=0\end{cases}$

27. Sol.(b)

Given that $f(x)=(\log_{\cos x}\sin x)(\log_{\sin x}\cos x)^{-1}+\sin^{-1}\left(\frac{2x}{1+x^2}\right)$

$$=\frac{\log_{\cos x}\sin x}{\log_{\sin x}\cos x}+\sin^{-1}\left(\frac{2x}{1+x^2}\right)=\left(\frac{\log\sin x}{\log\ \cos x}\right)^2+2\tan^{-1}x=u+v$$

So that $f'(x)=\frac{du}{dx}+\frac{dv}{dx}$

Nos,

$$\frac{du}{dx}=2\left(\frac{\log\sin x}{\log\cos x}\right)\left[\frac{\cot x\log\ \cos x+\tan x\log\sin x}{(\log\cos x)^2}\right]$$

$$\therefore\ \left.\frac{du}{dx}\right]_{x=\pi/4}=2\left(\frac{\log(1/\sqrt{2})}{\log(1/\sqrt{2})}\right)\times\left[\frac{1\log(1/\sqrt{2})+1\log(1/\sqrt{2})}{(\log(1/\sqrt{2})^2)}\right]$$

$$=-8\log_2 e$$

Also , $\frac{dv}{dx}=\frac{2}{1+x^2}$

$$\therefore\ \left.\frac{dv}{dx}\right]_{x=\pi/4}=\frac{2}{1+\frac{\pi^2}{16}}=\frac{32}{16+\pi^3}$$

$\therefore$ from (1), $f'(x)]_{x=\pi/4}=-8\log_2 e+\frac{32}{16+\pi^2}$.

28. Sol.(c)

$$f(e^+)=\lim_{h\to 0}(e+h-e).2^{-2^{\frac{1}{e-(e+h)}}}=\lim_{h\to 0}(h).2^{-2^{\frac{1}{h}}}=0\times 1=0\ \text{(as for } h\to 0,\ -\frac{1}{h}\to-\infty\Rightarrow\ 2^{\frac{1}{h}}\to 0)$$

$$f(e^-)=\lim_{x\to 0}(-h).2^{-2^{\frac{1}{h}}}=0\times 0=0$$

Hence $f(x)$ is continuous $x=e$.

$$f'(e^+)=\lim_{h\to 0}\frac{f(e+h)-f(e)}{h}=\lim_{h\to 0}\frac{h.2^{-2^{\frac{1}{h}}}-0}{h}=\lim_{h\to 0}2^{-2^{\frac{1}{h}}}=1$$

$$f'(e^-)=\lim_{h\to 0}\frac{f(e-h)-f(0)}{-h}=\lim_{h\to 0}\frac{(-h).2^{-2^{\frac{1}{h}}}-0}{-h}=\lim_{h\to 0}2^{-2^{\frac{1}{h}}}=0$$

Hence $f(x)$ is non –differentiable at $x=e$.

29. Sol.(d)

Let k be an integer.

$f(k)=0,\ f(k-0)=(k-1)^2-(k^2-1)=2-2k$

$f(k+0)=k^2-(k^2)=0$

If $f(x)$ is continuous at $x=k$, then $2-2k=0$ or $k=1$.

30. Sol.(a)

$g(x)=|(x-1)(x-2)(x-3)|$ is non-differentiable at $x=1,2,3$.

But for $h(x)=\sin\pi x,\ h(1)=h(2)=h(3)=0$

So, $f(x)$ is differentiable at $x=1,2,3$.

Therefore $f(x)$ is differentiable at all real values of x.